Finite Element Analyses of Salt Domes with Stored Hot Wastes

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ABSTRACT

Finite element models of salt domes with hot wastes stored at different sites within the domes are used to simulate the time dependent behavior of domes relative to their behavior without stored hot wastes. Domal salt behavior is represented as similar to that of a viscous fluid with temperature dependent viscosity. Slow-flow and temperature-distribution finite element programs are coupled to achieve the results of this study.

No attempt is made to predict the natural movement of salt domes. However, the relative accelerated movement of salt domes due to temperature increase associated with stored hot wastes is considered. Such potential movement is of considerable interest if domal salt is to be used for storage sites for hot, possibly radioactive, wastes.

The finite element models of salt domes employed herein are deterministic, as contrasted to numerical probabilistic models which have been used by other investigators to model the natural movement of salt domes. Material laws used to describe the temperature dependent behavior of salt are based in large part on previous experimental and numerical studies of rock salt pillar-model studies at L.S.U. and elsewhere.

The results of the finite element analyses indicate that, as time passes, a general trend in the behavior of the domes with stored hot wastes is to "mushroom" more at the top than similar domes without wastes. Thus, storage sites are brought closer to the surface of the domes.

INTRODUCTION

This study represents an initial effort in the application of the finite element method to analyze salt domes with stored wastes.

The projected accumulation of radioactive waste due to nuclear power reactors prompted the National Academy of Sciences to investigate the problem of storage of radioactive waste on land. In 1957 the committee issued its report. The conclusion reached by this committee was that the storage of radioactive waste in salt beds and salt domes offered possibly the most promising and practical solution of the problem.

As a result of this report further studies were initiated concerning the disposal of radioactive waste in salt cavities. These preliminary investigation culminated in an experiment called "Project Salt Vault" in which radioactive waste was stored in the Carey Salt Mine at Lyons, Kansas (Lomenick, 1968). The temperature and motion of the waste were monitored in the area of the experiment and throughout the mine. Experiments were also performed on salt pillar models in the Oak Ridge Laboratory to compare with the data obtained from the mine.

Much subsequent work, both theoretical and experimental has been done on the creep of electrically heated salt pillars. From this work a creep law for salt has been developed by workers in the Engineering Science Department at L.S.U. (Bergeron, 1968; Char, 1972; and Thoms et.al., 1972)

THE SALT DOME MODEL

Salt and surrounding sediments are assumed to behave through geologic time like very viscous fluids. Bouyancy due to the density difference between the salt and the surrounding media is believed to be the primary mechanism of growth, particularly in the larger domes. Another mechanism which may be a factor in the evolution of salt domes is that of downbuilding. Downbuilding occurs when the sediments surrounding the salt dome sink. In the type domes considered here upthrusting is assumed to be the primary mechanism of growth. Therefore, no effects due to downbuilding are considered.

In 1934, Nettleton performed his classic experiments by modeling the evolution of salt domes with fluids of different densities. His work contributed significantly to advance the bouyancy theory of salt dome evolution.

Kupfer (1966) has proposed a model to describe the horizontal migration and upward rise of salt into salt domes. It is upon this model that a suitable model for the present problem is based. Therefore, it would be proper to consider Kupfer's model in some detail. When salt beds reach some critical depth they begin to migrate laterally toward a center diaperism (Fig. 1). The salt then rises forming stocks. In some cases the basement has to rise to maintain isostatic balance. The salt is carried high into the surface sediments by the bouyancy of the column of salt below it. Because salt is denser than the surface sediments it tends to flow outward near the surface thereby producing the mushroom effect (Fig. 2).

In the latter stages of evolution the salt dome and the source may separate. If this is the case the salt dome will assume a teardrop shape (Fig. 3). As an example to illustrate this consider a drop of water falling through a less dense viscous fluid such as air. Kupfer has indicated in personnel communication that certain domes in Germany are thought to be of this shape.

In the following analysis hot waste will be simulated in an idealized salt dome model. The salt dome is in the latter stages of evolution. Therefore, it can be reasonably assumed that the salt dome has assumed a teardrop shape. Under the ideal conditions assumed here the salt dome model is axisymmetric, thus mathematically two dimensional. Also, because this analysis is concerned only with the effects of hot waste on the salt dome and not on the natural evolution of the salt dome, the reference frame chosen is attached to, and moving with, the idealized salt dome.

PROBLEM SOLUTION

A creep law for salt must be found in order to predict the relative motion of the salt dome with hot waste. It is assumed that salt, as well as sediments, behave in geologic time like very viscous fluids.

The creep law adopted for this problem is a modified

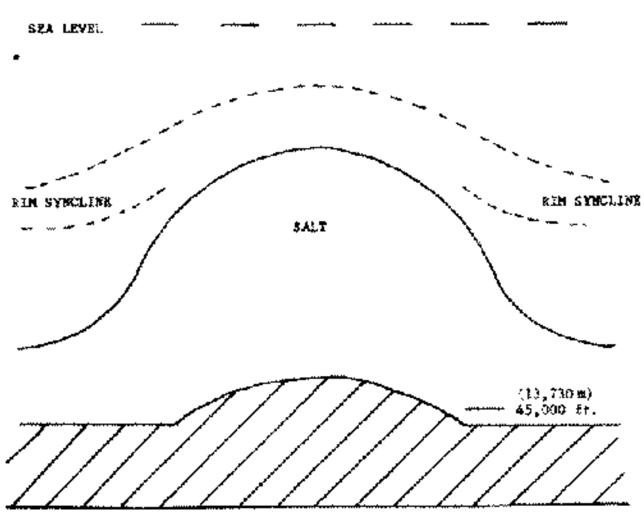


Figure 1. Start of Evolution.

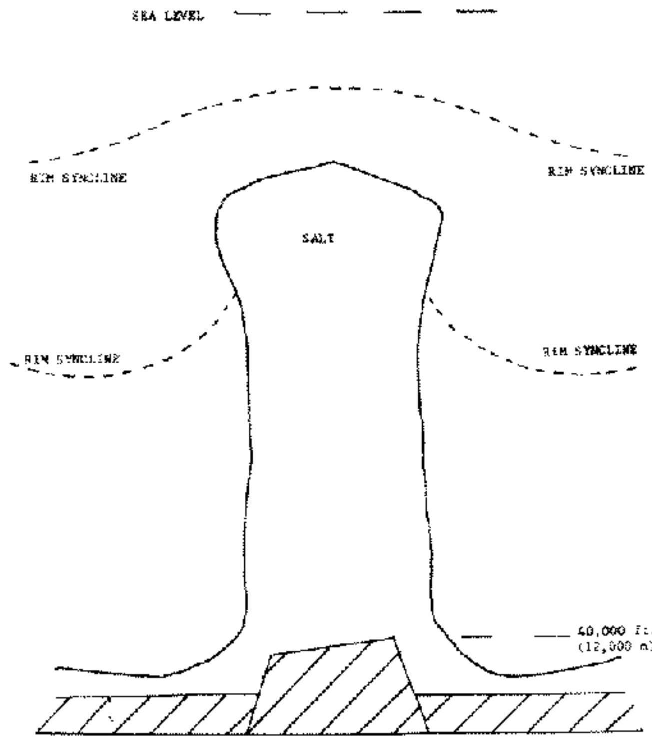


Figure 2. Maturing Salt Dome.

version of the creep employed at L.S.U. on salt pillar models. The creep law is,

$$\{\dot{\epsilon}\}_t = \Phi\ (J_2, t, T)\ [C_c]^{-1}\ \{\sigma\}_t \qquad (1)$$

thus approximately

$$\{\Delta\epsilon\} = \Phi\ (J_2, t, T)\ [C_c]^{-1}\ \{\sigma\}\Delta t \qquad (2)$$

where

$$\Phi = C_1\ J_2^{\ b_1}\ t^{b_2} T^{b_3}$$

$\{\epsilon\}, \{\dot{\epsilon}\}$ = strain vector, rate of strain vector

$\{\sigma\}$ = stress vector

J_2 = second invarient of deviator stress

T = temperature (degrees Kelvin)

t = time

Δt = increment of time associated with the time t (hr.)

$C_1 = \dfrac{.39 \times 10^{-36}}{7.5}$ $b_1 = 1.0$ $b_2 = -.75$ $b_2 = 9.65$

$$[C_c]^{-1} = \frac{(1-\nu)}{(1-\nu)(1-2\nu)} \begin{bmatrix} 1 & \frac{\nu}{1-\nu} & \frac{\nu}{1-\nu} & 0 \\ & 1 & \frac{\nu}{1-\nu} & 0 \\ & & 1 & 0 \\ & \text{(symmetric)} & & \frac{1-2\nu}{2(1-\nu)} \end{bmatrix}$$

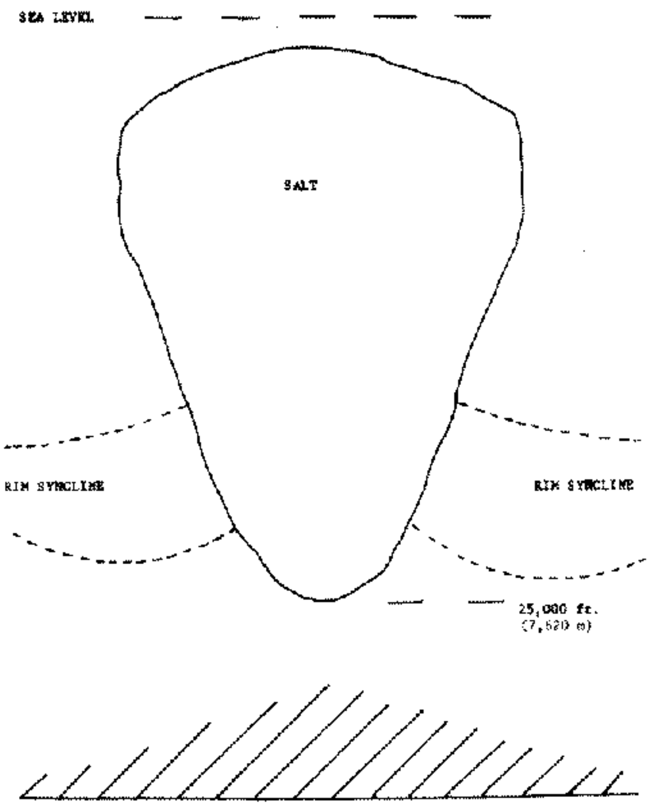


Figure 3. Detached Salt Dome.

Salt pillar analyses must consider the "strain hardening" of the salt. Therefore, the creep law employed in such analyses must contain, as it does, a time dependent term. In analyzing the salt dome it is assumed that the salt is in a steady state creep phase. Therefore, the modifications made to the above creep law must be the exclusion of the strain hardening term. This is done by replacing the time dependent term by a constant. The constant has been chosen to be 10,000 hours. This choice is based upon the results of experimental work by Char (1972) on salt from Weeks Island. After 10,000 hours the salt exhibits approximately steady state behavior. The variation of this constant will effectively change the time scale. Experimental work would have to be carried out on salt in place to determine the actual value of this constant necessary for the time to correspond to actual time.

The finite element method is used for the solution of this problem. The minimization of the rate of dissipation of mechanical energy can be used to generate a set of simultaneous algebraic equations which can be solved for the velocities of the nodal points. Dieterich and Onat (1969) were among the first to employ this principle in conjunction with the finite element method. The rate of dissipation of mechanical energy is,

$$-\int <\dot{\epsilon}> \{\sigma\}\, d(vol) + <\dot{\delta}> \{F\} \text{ at any time t} \qquad (3)$$

where vol = volume

$<\dot{\delta}>$ = displacement vector

$\{F\}$ = force vector

The minimization of this expression yields:

$$[V]\ \{\dot{\delta}\} = \{F\} \qquad (4)$$

where [V] = viscosity matrix for the system

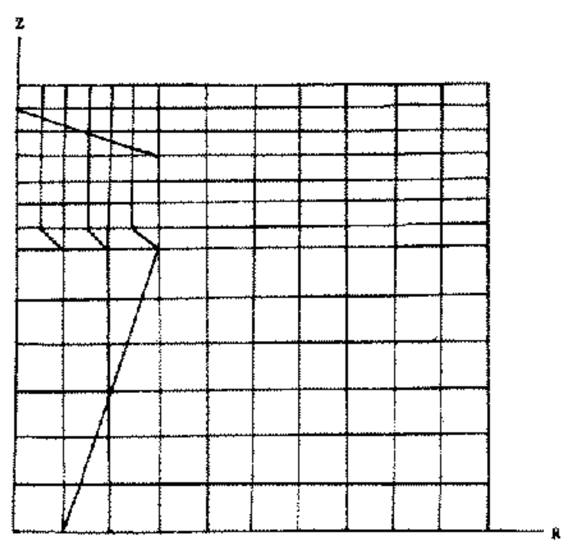
$\{\dot{\delta}\}$ = mean velocity vector

It should be noted that:

$$\{\delta\} \div \{\dot{\delta}\}\ \Delta t \qquad (5)$$

Within each element displacements are defined in terms of the displacement of the nodal points of the element. From the displacement function and the material properties of the element the strain and stress within the element can be determined.

The finite element method as incorporated in Wilson's heat conduction program (1966) is applied to calculate the temperature distribution in the salt. With the information available sufficient boundary conditions can not be postulated for the salt dome alone. Therefore, it is necessary to include the surrounding sediments in the salt dome model used to determine the temperature distribution (Fig. 4). The boundary is now taken to be the extremity of the surrounding sediment included in this model. The temperature of the nodes along the boundary are set equal to the ambient temperature of the earth at that particular depth. The assumption made in choosing this boundary is that



Scale: 1 in.=6000 ft.

Figure 4. Finite Element Model for Temperature Analysis.

these boundary nodes are so far removed from the waste that the waste has no effect on their temperature.

The temperature distribution throughout the salt dome can be calculated when no waste is present by setting the temperature on the boundary nodes and solving the steady state temperature distribution problem. The temperature of the waste is a function of time as well as the configuration and concentration of the waste in the salt dome. Due to the lengths of time involved and the thermal conductivity of the salt, the temperature of the waste can be approximated by a step function. Therefore, the temperature distribution in the salt dome can be approximated by the solution of a series of steady state temperature distribution problems.

Knowing the temperature distribution it is possible to apply the modified version of the creep law to modify Wilson's stress program (1956) so that it can be used to handle creep phenomena. For this portion of the analysis it was assumed that the salt dome alone could be modeled (Fig. 5).

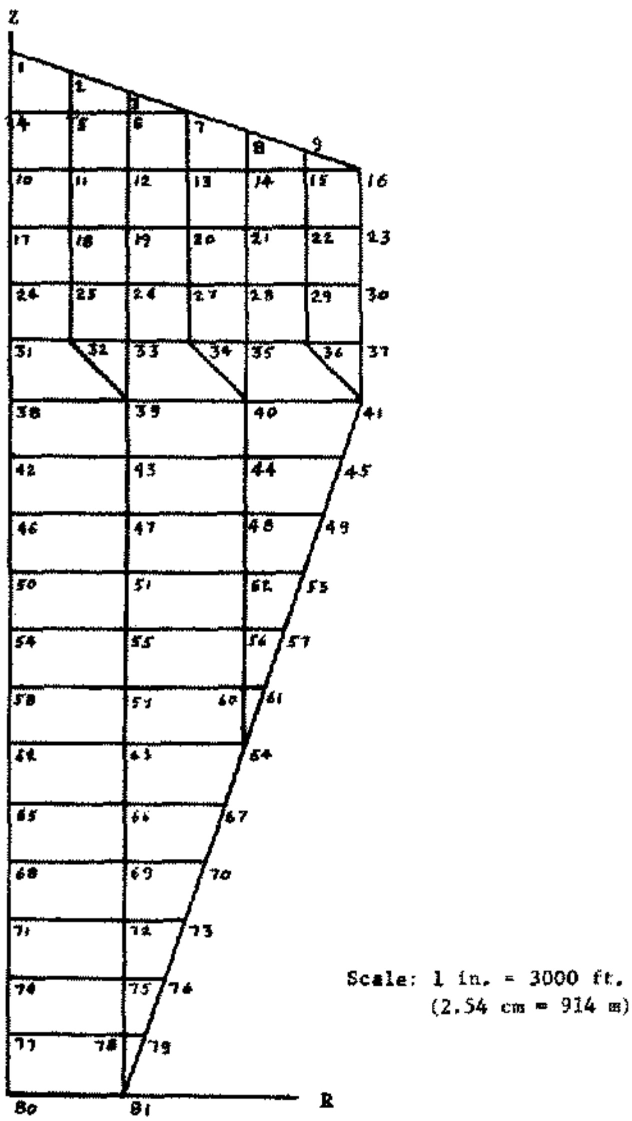


Figure 5. Finite Element Model for Creep Analysis.

All of the forces which act on the salt dome must be considered. These forces are separated into two categories, bouyant forces and friction forces. The bouyant force for each element is equal to the acceleration of gravity times the volume of the element times the density difference between the sediments and the salt at the depth of the centroid of the element. The friction force per unit area is taken to be a constant over the entire salt dome. The constant chosen is that necessary for the salt dome without waste to maintain, as nearly as possible, an equilibrium position. The friction forces are always directed to oppose an upward motion of the salt dome.

For a given time step, $\Delta t > 0$, a modulus is calculated for each element from the modified version of the creep law. This modulus may be referred to as the creep modulus. The creep modulus is a function of the state of stress of the element in the proceeding step, and the temperature of the element. Recall that the temperature distribution

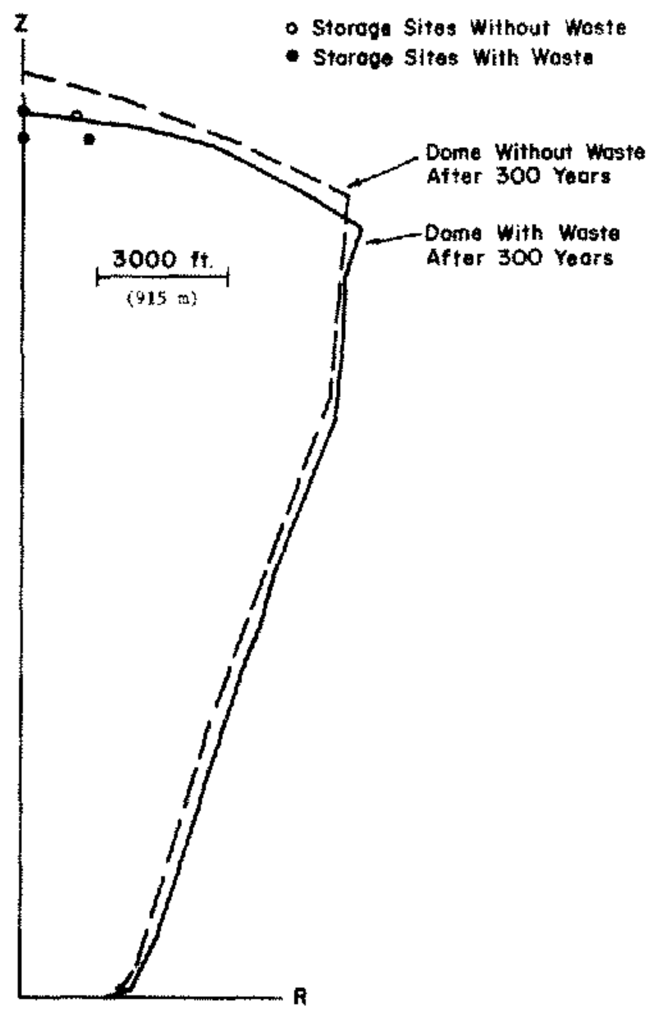


Figure 6. Relative Configurations of Dome With and Without Waste at Depicted Sites.

has been determined from Wilson's heat conduction program. The displacement of the nodes for the time step and the stresses in each element are calculated. The new coordinates of each nodal point are found by adding the displacements of the node for a given time step to the total previous displacements of that node. This analysis is carried out for the salt dome with and without waste.

RESULTS

The results shown in the following pages (Figs. 6–9) were obtained for a series of analyses with the waste stored at various sites. Based on the work of Birch (1958), the maximum increase in temperature at the location of radioactive waste disposal in a salt dome was taken to be 266°F. Birch, in determining the temperature of the waste, took into consideration the following factors: radioactive waste concentration, material and thermal properties of the surroundings, and radioactive waste configuration. The temperatures of the waste was allowed to remain at this maximum over the entire interval of interest so an upper bound on the displacement of the salt dome with radioactive waste would be obtained. A more refined analysis would be obtained by employing a stepwise variation of the temperature. In each case the time interval of interest was taken to be 300 years and the time steps were taken to be 10 years. It should be recalled that due to the somewhat arbitrary method of selecting the constant to replace the time dependent term in the creep law, the time considered is not necessarily the actual time. If, however, the time considered is in fact the actual time, this should be sufficient time for the temperature effects due to the radioactive wastes to subside. A time step of 10 years was found to yield results which compared favorably with results obtained using a smaller time step.

In Figures 6 through 9 the dashed lines represent the salt dome configuration without waste after 300 years have elapsed, the solid disks the position of the wastes in the salt dome with waste, and the hollow disks indicate these positions in the salt dome without wastes.

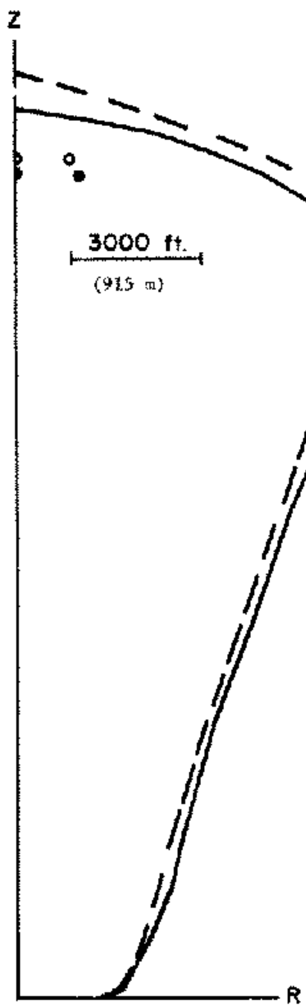


Figure 7. Salt Dome Without Waste and With Waste at Depicted Sites.

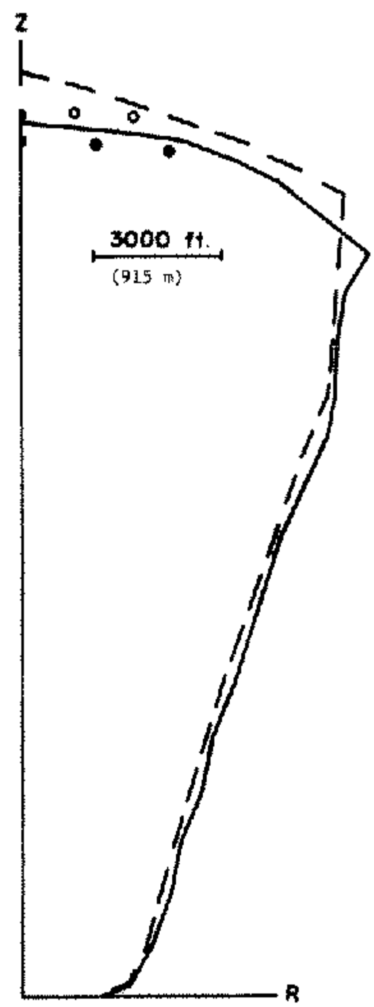


Figure 8. Relative Configurations of Dome With and Without Waste at Depicted Sites.

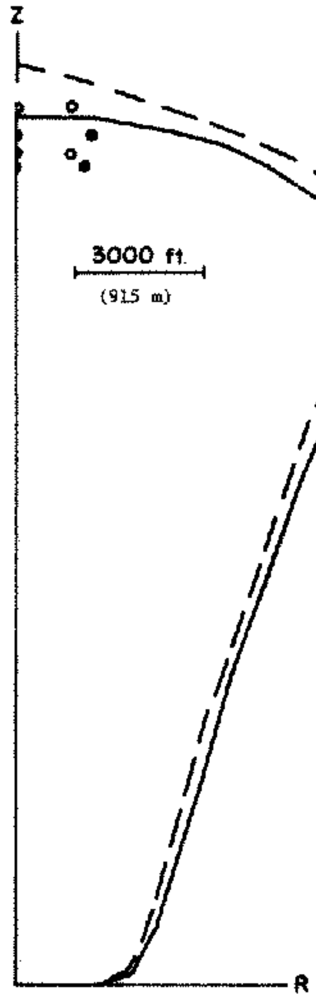


Figure 9. Relative Configurations of Dome With and Without Waste at Depicted Sites.

CONCLUSION

This study represents an initial effort in this research area. Further analysis of a salt dome is hindered by a lack of information on existing conditions associated with the in place salt dome. Although lack of information may hinder, it should not stop futher research into this problem.

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